

THEORETICAL AND EXPERIMENTAL STUDY OF WAVE
FORMS OF TRANSFORMER SUPPLYING
MERCURY-ARC RECTIFIER

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A THESIS

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by

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SYMBOLS USED

a_1	Instantaneous secondary current in transformer No. 1
a_2	Instantaneous secondary current in transformer No. 2
a_3	Instantaneous secondary current in transformer No. 3
E	Applied voltage to the transformers
E_{DC}	Rectifier output voltage
E_S	Secondary phase voltage of transformers
i_1	Instantaneous primary phase current in transformer No. 1
i_2	Instantaneous primary phase current in transformer No. 2
i_3	Instantaneous primary phase current in transformer No. 3
I_{DC}	Rectifier output current
I_L	Primary line current
I_ϕ	Primary phase current
I_S	Secondary transformer current
N_1	Primary transformer turns
N_2	Secondary transformer turns
p	Number of rectifier anodes
R	Load resistance
U.F.	Secondary utility factor
V.A.	Secondary volt-amperes
W_{AC}	Alternating current load resulting in same power loss per phase as produced by direct current load on the rectifier
W_{DC}	Direct current power output per phase of rectifier

TABLE OF CONTENTS

	PAGE
Approval Sheet	ii
Acknowledgment	iii
Symbols Used	vi
List of Figures	vii

CHAPTERS

I. INTRODUCTION	1
II. APPARATUS AND EXPERIMENTS	5
III. MATHEMATICAL ANALYSIS	9
IV. DISCUSSION OF CURRENT WAVE SHAPES	20
V. DISCUSSION OF CURVES	23
VI. CONCLUSION	27
BIBLIOGRAPHY	47

LIST OF TABLES

TABLE	PAGE
I. Test Data	28
II. Corrected Data	29
III. Calculated Data	30

LIST OF FIGURES

FIGURE	PAGE
1. Assembled Test Equipment	31
2. Primary Line Current at Full Load	32
3. Primary Line Current at 75% of Full Load	33
4. Primary Line Current at 46% of Full Load	34
5. Primary Phase Current at Full Load	35
6. Primary Phase Current at 75% of Full Load	36
7. Primary Phase Current at 46% of Full Load	37
8. Secondary Phase Current at Full Load	38
9. Secondary Phase Current at 75% of Full Load	39
10. Secondary Phase Current at 46% of Full Load	40
11. Secondary Neutral Current at Full Load	41
12. E_S vs I_{DC}	42
13. I_S vs I_{DC}	43
14. I_ϕ vs I_{DC}	44
15. I_L vs I_{DC}	45

THEORETICAL AND EXPERIMENTAL STUDY OF WAVE

FORMS OF TRANSFORMER SUPPLYING

MERCURY-ARC RECTIFIER

CHAPTER I

INTRODUCTION

The conversion of electrical energy from alternating current to direct current is the fundamental purpose of a rectifier. The flow of electricity through the tube of a mercury-arc rectifier is by a stream of electrons that have been liberated from the cathode, which is the mercury pool, traveling to the graphite anode. Since the electrons are negatively charged, they will always pass toward a more positively charged electrode than that from which they came. Thus, the flow of current will always flow through the tube when the anode is more positive than the cathode for on the reversal of the input voltage the graphite cannot give off electrons because that is not one of its qualities.

In a rectifier of this type the alternating current that is present at the input of the rectifier is modified by the suppression of one of the two directions of pulses to give only unidirectional current from the rectifier. This unidirectional current is, therefore, pulsating, and can be analyzed as a steady direct current with a superimposed alternating current.

The mercury-arc rectifier was first recognized by Jemin and Menouvrier, in 1882, when they discovered that an electric arc could be established between electrodes made of mercury and carbon. From these discoveries came one of the first mercury-vapor tubes, which was built by Arons. In 1889 Fleming made an investigation of the unilateral conductivity of the electric arc in air. These experiments were followed by those of Sahulka, in 1894 and 1898, when he made studies of atmospheric areas established between mercury and carbon.¹

Although all of these studies were to have an important bearing on future work, the idea of utilizing the unidirectional arc for practical purposes was not considered. Cooper-Hewitt began manufacturing mercury-vapor lamps toward the end of the nineteenth century, and it was then that the first mercury-arc rectifiers were constructed. These original rectifiers were of the glass tube type, of small capacity only. Many problems of design, such as conductor connections and vacuum seals between metal and glass, were overcome. As these difficulties were eliminated, larger and more efficient rectifiers were manufactured; then with greater power demands, the steel-tank rectifier was introduced. This new type has now replaced the glass tube model where the power output must be large, and is widely used as a replacement for the rotary converter.

An even more modern development in mercury-arc rectifiers is the Ignitron Rectifier. This rectifier consists of an assembly of steel-

¹H. Rissik, Mercury-Arc Current Converters (London: Sir Isaac Pitman & Sons, Ltd., 1935), p. 1.

tanks or tubes in which there is a graphite anode and a mercury cathode. The principle of this rectifier is the provision of a method for starting the arc in a few micro-seconds. The cathode spot for each cycle is ignited by a high resistance rod which is mounted so that it is partially immersed in the mercury pool. When a small positive potential is applied to this ignitor rod, a current in the order of 15 amperes flows and creates the spot. If the anode is more positive than the cathode, the anode will pick up current. During the period that the applied voltage is reversed, the cathode spot will go out, leaving the anode surrounded by deionized gas. This is caused by a deionizing baffle between the cathode and the anode. This elimination of the ionized gas during the period of non-conduction removes the condition favorable to the flow of current in the reverse direction, i.e., arc-back.

David C. Prince and Francis B. Vogdes², in 1927, published a book on the mercury-arc rectifier. In this book they attempted to show by mathematics what should happen, theoretically, to the currents and voltages both in the rectifier and in the transformers feeding the rectifier.

The purpose of this thesis is to work out the formulas published by Prince and Vogdes as they regard transformers connected delta on the primary side and star on the secondary and, secondly, to compare theoretical results obtained from their work with those actually obtained from experimental tests. These experiments were made in the Industrial Electronics Laboratory of the Georgia School of Technology. In order to

²David C. Prince and Francis B. Vogdes, Principles of Mercury Arc Rectifiers and Their Circuits (New York: McGraw-Hill Book Company, Inc., 1927.)

make tests with a polyphase circuit it was necessary to use an Ignition Rectifier. The results obtained from the use of this rectifier as compared to those obtained from any other type polyphase mercury-arc rectifier would not differ as far as the purpose of this work is concerned.

CHAPTER II

APPARATUS AND EXPERIMENTS

The equipment used in the conduction of this work is shown in Figure 1, page 31, with the exception of three induction voltage regulators, which were used to maintain the supply voltage to the primary of the transformers at a constant value.

The transformers used were three 2.5 kva, 220/220 Westinghouse connected delta on the primary side and star on the secondary. Interconnectors between the transformers, line, and rectifier were made with heavy copper cable of the same length, in order to keep the transformers balanced as closely as possible. In this way all photographs could be made from one transformer since the other two were identical.

A Westinghouse Mercury-Arc Ignitron Rectifier was used as the rectifier for the experiments. This converter is rated to supply 75 amperes and 250 volts DC. It is here pointed out that the transformers with a bank rating of 7.5 kva were much too small to supply the rectifier, but in the absence of any other suitable transformers these were used with 150% overload for short periods. Even with this overload condition the rectifier could only be loaded 46.7% of its normal full load current.

This rectifier consisted of three tubes connected to form one common cathode for the assembly. The cathode, which is the positive DC terminal, was connected to the load. The negative terminal of the rectifier was connected to the neutral of the three transformers con-

nected in star.

During operation the tubes of the rectifier were kept cool by pumping water through coils built around the outside of the tubes. The rate of flow of the cooling liquid was regulated by means of a gate valve located in the rectifier base.

The electrical networks, contained in the rectifier for purposes of phase shifting and ignition, have no bearing on the subject of this thesis and will not be further discussed.

A heavy duty, pure resistance was used to load the rectifier. This variable resistor was connected in series to the neutral of the transformer secondaries and the common cathode of the rectifier. Due to the limitations involved in using small transformers only about half of the resistance settings could be used; however, the settings were of such values to give even increments over the load range.

Referring to Figure 1, the table above the transformers held most of the meters used to record the results of the load tests. Volt meters were placed across the primary lines, secondary legs, and across the load. The supply voltage was maintained constant at 220 volts. Ammeters read the primary line and phase, secondary, and DC load currents.

Since the secondary current was pulsating direct current, dynamometer type ammeters were used to record the root-mean-square value of this current. Because the dynamometer type ammeters available could only read up to ten amperes full scale deflection, it was necessary to use three such ammeters connected in parallel for maximum loading of the rectifier. At loads of less than twenty

amperes, one of the ammeters was removed from the circuit. Similarly, for loads of less than ten amperes, two ammeters were removed. In this way it was possible to make more accurate readings.

Wattmeters were connected in the primary circuit to indicate the power input. In all cases, the meters used were of the best quality, very carefully calibrated from standard meters to give true and accurate values.

Photographs of the current waves were made from an oscilloscope connected in various networks to give the desired picture. Identical resistances of low value were placed in the primary lines, primary phases, secondary legs, secondary neutral, and direct current load circuit. Thus the voltage drop across any of these resistances would be an indication of the current. Then the oscilloscope, when connected across the resistances, would reveal not only the current but also, the wave shape of the current. The amplification control of the oscilloscope was held constant for the entire load test so that the relative magnitudes of the load currents could be observed at different settings of the rheostat.

Before tests were made, the equipment was turned on and allowed to warm up for thirty minutes. The load was then varied in eleven steps; from the lightest possible load, i.e., that point where the tubes first ignited and continued to do so without skipping, to the value of that resistance which produced 150% overload for the transformers. Meter readings were recorded at each separate setting of the load resistor. At positions of 100%, 75%, and 46% of full load, photographs were made of the primary line current, primary phase cur-

rent, and secondary leg current. A picture was also made of the neutral current at full load. These photographs are reproduced in Figures 2 - 11.

Some difficulty was encountered in the determination of the arc drop across the tubes. However, after unsuccessful results with a vacuum tube arrangement, the arc drop was measured as follows: The rectifier was set in operation in the normal method. Next, a DC voltage supply, connected in series with a variable resistance, was placed between anode and cathode, positive to anode and negative to cathode, of the tube to be measured while the AC voltage was still being supplied. Then the AC line switch was opened, disconnecting the AC supply to the transformers. The DC load was then opened; this left a series circuit consisting of one rectifier tube, a variable resistance, and a DC supply. Since the polarity of the DC source was applied properly, the tube never ceased to fire even after the AC source was disconnected.

Now that the tube was firing continuously due to the DC supply, a DC voltmeter placed between anode and cathode indicated the arc drop for that tube. In this manner of measuring the arc drop the problem of back voltages was eliminated. All of the tubes gave an arc drop voltage of approximately 9.6 volts over the range of operation used in this work. Therefore that value was used in the correction of test data.

CHAPTER III

MATHEMATICAL ANALYSIS

The fundamental basis of operation of the mercury-arc rectifier is the passage of current in one direction and the prohibition of current flow in the opposite direction. A study of the circuits cannot be attempted without first analyzing the wave forms of the particular system to be analyzed.

Several assumptions are made in order to reduce the magnitude of the problem. First, the output of the rectifier is assumed to be a pure direct current with no fluctuations which, in time, will cause the anode currents to be of rectangular shape. Second, the arc drops of the tubes themselves are neglected since it would be impossible to arrive at a set value for all rectifiers. Third, all transformer losses are neglected, as are the effects of reactance and magnetic leakage. Fourth, the alternating current supply voltage is considered to be sinusoidal under all conditions of rectifier loading. Finally, the magnetizing current of the transformers feeding the rectifier is so small that it is neglected in the analysis.

The assumption that the rectifier output is a direct current of square wave shape will cause the results obtained mathematically to differ somewhat from those actually found by experiment, because, in the operation of the rectifier, a pure resistance was used to load the rectifier and no filtering action took place. The other assumptions will have little effect on the mathematical results obtained,

other than to simplify the mathematics involved.

This thesis deals only with mercury-arc rectifiers fed by transformers whose primaries are connected in delta and secondaries in star. With this connection only half wave rectification can be achieved. Current flows in each phase of the secondary for one-third of a cycle, or $2/3 \cdot \pi$ radians, while during this time the direct current output of the rectifier is constant. The voltage across the load is the same as that of the anode which is in the process of firing. Thus the direct voltage is the average of a sine wave for one-third of a cycle, or $2/3 \cdot \pi$ radians. This may be represented in mathematical form by the equation

$$E_{DC} = \sqrt{2} E \frac{3}{2\pi} \int_{-\frac{\pi}{3}}^{+\frac{\pi}{3}} \cos \theta \, d\theta \quad (1)$$

On performing the indicated integration and substituting the limits, (1) becomes

$$E_{DC} = \sqrt{2} E \frac{3}{\pi} \sin \frac{\pi}{3} \quad (2a)$$

$$= 1.169 E \quad (2b)$$

If the direct current output of the rectifier is I_{DC} , then from definition the root-mean-square value, or effective value, of the secondary and anode current is

$$I_S = \frac{I_{DC}}{\sqrt{3}} \quad (3)$$

The power output of the rectifier is $I_{DC} E_{DC}$, and the power loss per secondary phase will be $I_{DC}^2 R/3$. If W_{AC} is the alternating current load resulting in the same loss, then

$$\frac{I_{DC}^2 R}{3} = \left(\frac{W_{AC}}{E} \right)^2 R \quad (4)$$

Solving for W_{AC} :

$$W_{AC} = \frac{I_{DC} E}{\sqrt{3}} \quad (5)$$

The ratio of the rectifier power output to this alternating current power load has been designated the secondary utility factor and may be represented as

$$U.F. = \frac{W_{DC}}{W_{AC}} = \frac{I_{DC} E_{DC}}{\frac{I_{DC} E}{\sqrt{3}}} \quad (6)$$

Substituting the value of E_{DC} as obtained from (2a), (6) becomes

$$U.F. = \frac{\sqrt{2} E \frac{3}{\pi} \sin \frac{\pi}{3}}{\frac{E}{\sqrt{3}}} \quad (7a)$$

$$= \frac{\sqrt{2} \cdot \sqrt{3}}{\pi} \sin \frac{\pi}{3} \quad (7b)$$

$$U.F. = 0.675$$

(7c)

It is of interest to point out that the maximum secondary utility factor occurs when there are theoretically 2.69 anodes. For any number of phases the average direct potential is given by

$$E_{DC} = \sqrt{2} E \frac{p}{2\pi} \int_{-\frac{\pi}{p}}^{+\frac{\pi}{p}} \cos \theta \, d\theta \quad (8)$$

Integrating and simplifying (8), the equation reduces to

$$E_{DC} = \sqrt{2} E \frac{p}{\pi} \sin \frac{\pi}{p} \quad (9)$$

Then the DC power output for any number of phases is

$$W_{DC} = \sqrt{2} E I_{DC} \frac{p}{\pi} \sin \frac{\pi}{p} \quad (10)$$

The secondary power loss will be the $I_{DC}^2 R$ loss. The loss per phase is then W_{DC}/p , or $I_{DC}^2 R/p$. Since W_{AC} represents the alternating current load which will give the same power loss, a representation for W_{AC} may be derived in terms of the direct current output, and is

$$\left(\frac{W_{AC}}{E} \right)^2 R = \frac{I_{DC}^2}{p} R \quad (11)$$

Solving for W_{AC}

$$W_{AC} = \frac{I_{DC} E}{\sqrt{P}} \quad (12)$$

The utility factor for any number of anodes is then

$$\frac{W_{DC}}{W_{AC}} = U.F. = \frac{\sqrt{2} E I_{DC} \frac{\sin \frac{\pi}{P}}{\pi}}{\frac{I_{DC} E}{\sqrt{P}}} \quad (13a)$$

$$= \frac{\sqrt{2} \cdot \sqrt{P}}{\pi} \sin \frac{\pi}{P} \quad (13b)$$

To determine the value of P that will give the maximum utility factor, differentiate (13b) and set the differential equal to zero, thusly

$$\frac{d(U.F.)}{dP} = \frac{\sqrt{2}}{\pi} \left[-\sqrt{P} \frac{\pi}{P^2} \cos \frac{\pi}{P} + \frac{1}{2} \cdot \frac{1}{\sqrt{P}} \sin \frac{\pi}{P} \right] = 0 \quad (14)$$

Since P is obviously not zero the factor $\frac{1}{\sqrt{P}}$ may be divided out, (14) becomes

$$-\frac{\pi}{P} \cos \frac{\pi}{P} + \frac{1}{2} \sin \frac{\pi}{P} = 0 \quad (15)$$

Solving for p by Newton's Method, it is found that for maximum utility factor³

$$p = 2.69$$

Rectifiers must have a number of anodes that is an integer. Hence, the highest utility factor is actually obtained when the rectifier has three anodes.

Marti worked out mathematically the basic equations for the wave shape of the primary current for a six-phase rectifier with three-phase star connected transformer primaries under ideal conditions.⁴ The same line of reasoning may be applied for the straight delta-star connected transformers to obtain the wave shape which appears in the publication of Prince and Vogdes. It is emphasized that this derivation holds true only for three-phase transformers.

With the assumption of zero magnetizing magneto motive force, the sum of the m.m.f.s on the three legs of the transformer core, three-phase transformer, are equal to each other. The following equations may be written for the equality of the magneto motive forces:

$$N_1 i_1 + N_2 a_1 = N_1 i_2 + N_2 a_2 = N_1 i_3 + N_2 a_3 \quad (17)$$

³David C. Prince and Francis B. Vogdes, Principles of Mercury Arc Rectifiers and Their Circuits (New York: McGraw-Hill Book Company, Inc., 1927.) p. 96

⁴Othmar K. Marti and Harold Winograd, Mercury Arc Power Rectifiers (New York: McGraw-Hill Book Company, Inc., 1930.) pp.52-53

The sum of the three primary currents is equal to zero:

$$i_1 + i_2 + i_3 = 0 \quad (18)$$

For simplicity in solving these equations the ratio of transformation is assumed to be 1 to 1. This assumption has no effect on the results, since it only concerns the amplitude of the primary wave in relation to the secondary, but does not effect the relationship between phases.

Then, from equation (18)

$$i_1 + a_1 = i_2 + a_2 \quad (19)$$

$$i_1 + a_1 = i_3 + a_3 \quad (20)$$

Adding equation (19) and (20)

$$2 i_1 + 2a_1 = i_2 + i_3 + a_2 + a_3 \quad (21)$$

Utilizing equation (18), (21) reduces to

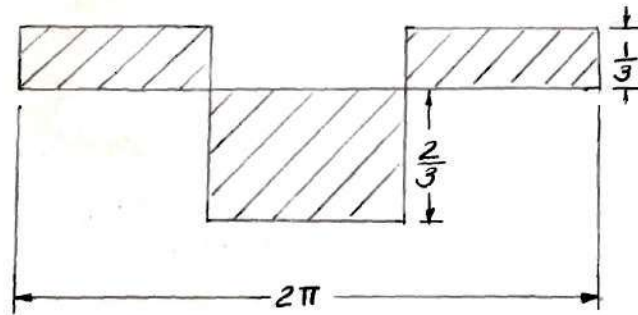
$$2 i_1 + 2a_1 = -i_1 + a_2 + a_3 \quad (22)$$

Collecting terms, (22) becomes

$$3i_1 = a_2 + a_3 - 2a_1 \quad (23)$$

$$i_1 = \frac{1}{3} a_2 + \frac{1}{3} a_3 - \frac{2}{3} a_1 \quad (24)$$

Equation (24) represented graphically:



It is necessary to emphasize the fact that the results of the above derivation cannot apply to a straight delta-star connection. With the transformer primaries connected delta and the secondaries star, load current can flow only in the primary of the transformer whose secondary is supplying current to a rectifier anode that is actually firing. Thus, during periods when an anode is inactive, no current flows in the secondary, and only magnetizing current is present in the corresponding primary winding. The primary phase current for single phase transformers arranged delta-star without zig-zag secondaries should be theoretically a rectangular block flowing for a period of $2\pi/3$ radians, and having no load current for the remaining portion of a complete cycle. The actual wave obtained will be discussed in the following chapter.

The secondary volt-amperes are

$$V A = 3 E I_{\phi} \quad (25)$$

Substituting for E , in terms of E_{DC} , that value obtained from equation (2b), and for I_{ϕ} , in terms of I_{DC} , its equivalent from equation (3), equation (25) becomes

$$V_A = 1.481 E_{DC} I_{DC} \quad (26)$$

The effective primary current is equal to the secondary current divided by the ratio of transformation if the transformers are connected delta-star. However, at this point Prince's and Vogdes' work differs considerably for this transformer connection and, although it is not stated in their publication, the results they obtain can only hold true for transformers whose secondaries are connected zig-zag.

The following derivation will be based on the supposition of a delta connected primary and an interconnected star secondary. This is done to arrive at the same conclusions as those of Prince and Vogdes.

The purpose of a zig-zag connected secondary where the transformers are feeding a mercury-arc rectifier is to eliminate saturation of the transformer cores by the direct currents carried in secondaries. With this connection the primary carries current closely matching that carried by the corresponding secondary while it is feeding an anode that is firing. Then when the next anode fires, identical current flows through the same primary, but in the opposite direction. The heating effect of such a current will be twice that of a current consisting of only the pulses in one direction and its root-mean-square value will be 2 times that of the unidirectional current. Thus the primary windings

must be able to carry an alternating current load $\sqrt{2}$ times the load which either secondary would carry.

A vector analysis of the zig-zag connection shows that the current in the secondary causes current to flow in two primary phases simultaneously with 60° phase difference. Therefore the current in one primary phase will be the total primary phase current divided by $\sqrt{3}$.

Taking the above into consideration the primary phase current in a single transformer can be represented by

$$I_\phi = \frac{\sqrt{2}}{3} I_{DC} \quad (27)$$

The line current is equal to the primary phase current multiplied by $\sqrt{3}$.

$$I_L = \frac{\sqrt{2}}{\sqrt{3}} I_{DC} \quad (28)$$

The primary voltage in terms of the direct current voltage because of the one to one ratio of transformation is the same as the secondary voltage. Therefore from equation (9)

$$E = \frac{E_{DC}}{\sqrt{2} \times \frac{3}{\pi} \sin \frac{\pi}{3}} \quad (29)$$

For delta-star connected transformers with the secondaries not interconnected, the primary phase current should be the same as the

secondary current, equation (3). The primary line current would then be the same value as the load current, I_{DC} .

CHAPTER IV

DISCUSSION OF CURRENT WAVE SHAPES

It has been previously stated that photographs were made of the currents in the various transformer circuits for conditions of 100%, 75%, and 46% of full load. Full load has been defined as that direct current load which caused 150% of normal rated transformer current to flow in the primaries of the transformers feeding the mercury-arc rectifier. These photographs have been reproduced in Figures 2-11 of this thesis.

Figures 2, 5, and 8 represent a complete set of photographs for the full load values of the currents in the transformer primary line, primary phase, and secondary phase, respectively. Figure 8 shows the secondary phase current of one transformer through two firing periods. Instead of the ideal square block current wave, which was considered for the mathematical analysis, it is seen that, while the sides of the pulse are relatively steep, the top of the wave follows that of a sine wave. This is caused by the fact that the load is a pure resistance and, since the applied voltage is virtually a sine wave, the current has a shape closely resembling that of the voltage.

Figure 5 represents the picture of the primary phase current for the same transformer. Here the reflection of the secondary load current is clearly seen as the first negative lobe, and has practically the same shape as the secondary pulse. Since there can be no direct

current component in the primary resulting from the direct current in the secondary, a positive pulse must be present to counterbalance the load impulse. This positive lobe is apparently due to the partial saturation of the transformer core by the flow of direct current in the secondary. Under this condition the transformer is operating higher on its magnetization curve and a larger magnetizing current is required to produce the necessary flux. A comparison of the average values of the positive and negative pulses shows the absence of any DC component of primary phase current.

Figure 2, illustrating the wave form of the primary line current, is obtained by vectorily adding the phase currents of two transformers whose primaries are being fed by the same line. Since these currents are displaced from one another by 120 electrical degrees, the load impulse of one transformer adds directly to the magnetizing impulse of the other, giving the large negative lobe seen on the photograph. The first positive pulse consists only of the magnetizing current of transformer No. 1 (so called for convenience of identification) because transformer No. 2 is producing negligible current to the line as seen from the horizontal trace on Figure 5. The second positive lobe is the load impulse of transformer No. 2 for transformer No. 1 is by this time carrying a very small magnetizing current in the reverse direction. The load current of transformer No. 1 and the magnetizing current of transformer No. 2 again add to give the second large negative pulse.

Figures 3, 6, and 9 are similar to the ones discussed above with this exception: the load has been reduced to 75% of full load. Since

the direct current value has been reduced, the transformer is not operating as high on its magnetization curve as it was for full load and, consequently, the magnetizing current will not be as great. Figure 6 shows that the positive lobe has decreased sufficiently to conform to the reduction of the load current impulse. Similarly, Figures 4, 7, and 10 apply for loading of 46% of full load.

Figure 11 is a photograph of the secondary neutral current at full load. From the oscilloscope trace it is evident that there is a slight period of overlap and, to further illustrate this fact, the phase currents have been extended in ink. This period of overlap represents an instant of short circuit between the two secondary phases involved. The affect of this overlap on the output voltage of the rectifier is discussed in the following chapter.

CHAPTER V

DISCUSSION OF CURVES

In the ideal cases, as considered by Prince and Vogdes, the power factor is assumed to be unity. Obviously there will result some degree of error from this assumption, and the amount of error will vary in different cases. Therefore, to be in accord with this supposition, the test data obtained in the laboratory experiments has been corrected. The new value of primary current is that which would flow at unity power factor to give the same power input as indicated by the wattmeter readings.

With these values of corrected data a curve, Figure 12, was plotted from test data with the rectifier output load current, I_{DC} , the abscissa and the secondary voltage to neutral, E_S , the ordinate. These points for the curve, labeled "Measured", were obtained from a voltmeter connected between the neutral and the cathodes of the rectifier. A second curve was plotted on the same sheet with the above indicating the theoretical value of E_S under ideal conditions. It is again pointed out that the actual readings of the voltmeter across the load have been increased by 9.6 volts to eliminate the error of arc drop, a factor not considered by Prince and Vogdes. With the corrected values of E_{DC} , equation (2b) was applied, and the mathematical value of E_S was used in the curve indicated as "Calculated."

Examination of these two curves shows that at the lighter values

of load the difference in magnitudes is small, but, as the load is increased, the margin of difference increases. This fact may be attributed partly to the increase in drop in the output voltage caused by the resistance in the circuits as the load is increased. Also, a drop in voltage will result from the effects of inductance in the circuits. Up to this point the output current wave has been considered to be a square block, each anode carrying current $\frac{2\pi}{3}$ radians. Since it is impossible for the current flowing in a particular anode to start and stop instantaneously, it is necessary for current to flow simultaneously through two anodes for a brief instant. When this condition exists, there is temporarily a short-circuit, since both anodes are carrying current. Thus, the ends of the secondary windings are connected. This results in a distortion of the secondary voltage E_S . Since the output voltage is equal to the average of the instantaneous voltage of the conducting anode from the time one starts until the next one fires, any distortion will effect the output voltage. The short circuit condition of the secondary will cause the output voltage to be reduced.

Since the "Calculated" curve is obtained from the application of equation (2b) on the corrected output voltage, it is apparent that it will fall below the actual measured E_S .

In Figure 13, curves were plotted with I_S vs I_{DC} where I_{DC} remained as the abscissa and I_S was made the ordinate. A comparison of these curves shows a very close likeness between them since the assumptions made, having little effect on the rectifier and the calculated values, merely involved the division of the load current, I_{DC} ,

by $\sqrt{3}$. It must be said, however, that the use of $\sqrt{3}$ is not strictly correct. There is some overlapping of the anode currents; each does not carry current for $1/3$ of the complete cycle but for approximately 0.4 of the period, as indicated by the photograph of the secondary neutral current, Figure 11.

In passing from the transformer secondary to the primary, considerable differences are encountered. Curves showing the relation between the primary phase currents, I_ϕ , for both calculated and test values, as plotted against the output load current, I_{DC} , are shown in Figure 14. Here, the test current has been corrected for unity power factor in accordance with the basic assumptions made in the mathematical analysis, the results showing the calculated curve to be far beneath the one obtained by test. This results from the fact that the transformers used in these experiments were three single-phase transformers, while Prince and Vogdes evidently used transformers with interconnected secondaries to obtain their results. If that is true, with the connections and equipment used in these experiments, equation (27) is not applicable. Instead of the values used for plotting the "calculated" curve, the curve should be identical with the "calculated" curve of Figure 13, since the values should mathematically be the same.

Figure 15 is a graph of the measured and calculated values of the primary line current, I_L , plotted against the output load current, I_{DC} . These curves are very similar to the ones in Figure 14. The calculated values are the same except for the fact that they have all been multiplied by the factor $\sqrt{3}$. The measured values are very nearly $\sqrt{3}$

times those obtained from the ammeter readings of the primary phase currents.

From the above results it has been shown that the mathematical results recorded by Prince and Vogdes hold true only for transformers connected delta on the high side and zig-zag on the low-voltage side. These results cannot be applied to any other connection of transformers without special considerations and changes.

CHAPTER VI

CONCLUSION

In the foregoing work it has been proved that the analysis of Prince and Vogdes of a mercury-arc rectifier fed by three single-phase transformers connected straight delta-star is not correct as it appears in their publication. Their mathematical derivations are true only for transformers whose secondaries are interconnected, or zig-zag. The primary wave shapes for both line and phase are correct for a three-phase transformer connected delta on the primary and star on the secondary, and for three single-phase transformers connected delta-zig-zag, but their wave shapes are incorrect for single-phase transformers wired straight delta-star. These conclusions were borne out in the mathematical analysis and the photographs of the actual waves obtained from tests.

This thesis did not undertake to analyze mathematically the waves from the standpoint of harmonics and transient conditions. Obviously that would be a lengthy task; a subject deserving intensive study and a thesis for itself.

TABLE I

TEST DATA

E	E _S	E _{DC}	I _L	I _φ	I _S	I _{DC}	Total Watts Input
220	215	234	7.38	4.60	2.65	4.4	1,410
220	214	229	12.24	7.96	4.96	8.5	2,720
220	213	224	19.20	11.34	7.35	12.5	4,660
220	213	220	24.50	15.00	9.49	16.1	6,200
220	212	216	30.00	17.75	11.58	19.7	7,850
220	211	212	35.00	20.05	13.53	23.0	9,340
220	208	208	40.00	23.05	15.46	26.3	11,870
220	207	204	44.00	25.30	17.19	29.2	12,200
220	207	200	49.00	28.70	19.10	32.7	13,720
220	205	195	53.00	31.10	21.30	35.0	14,880

TABLE II

CORRECTED DATA

E	E _S	E _{DC}	I _L	I _φ	I _S	I _{DC}	Total Watts Input
220	215	243.6	3.70	2.30	2.65	4.4	1,410
220	214	235.6	7.14	4.65	4.96	8.5	2,720
220	213	233.6	12.25	7.24	7.35	12.5	4,660
220	213	229.6	16.25	9.95	9.49	16.1	6,200
220	212	225.6	20.60	12.20	11.58	19.7	7,850
220	211	221.6	24.50	14.02	13.53	23.0	9,340
220	208	217.6	28.32	16.31	15.46	26.3	11,870
220	207	213.6	32.00	18.40	17.19	29.2	12,200
220	207	209.6	36.10	21.10	19.10	32.7	13,720
220	205	204.6	39.10	22.93	21.30	35.0	14,880

TABLE III
CALCULATED DATA

E	I_L	I_ϕ	I_S
208	3.19	2.08	2.54
204	6.95	4.00	4.91
200	10.22	5.89	7.22
196	13.10	7.60	9.30
193	16.10	9.30	11.38
189	18.80	10.84	13.30
186	21.48	12.40	15.20
183	23.85	13.78	16.88
179	26.72	15.42	18.90
175	28.60	16.50	20.20

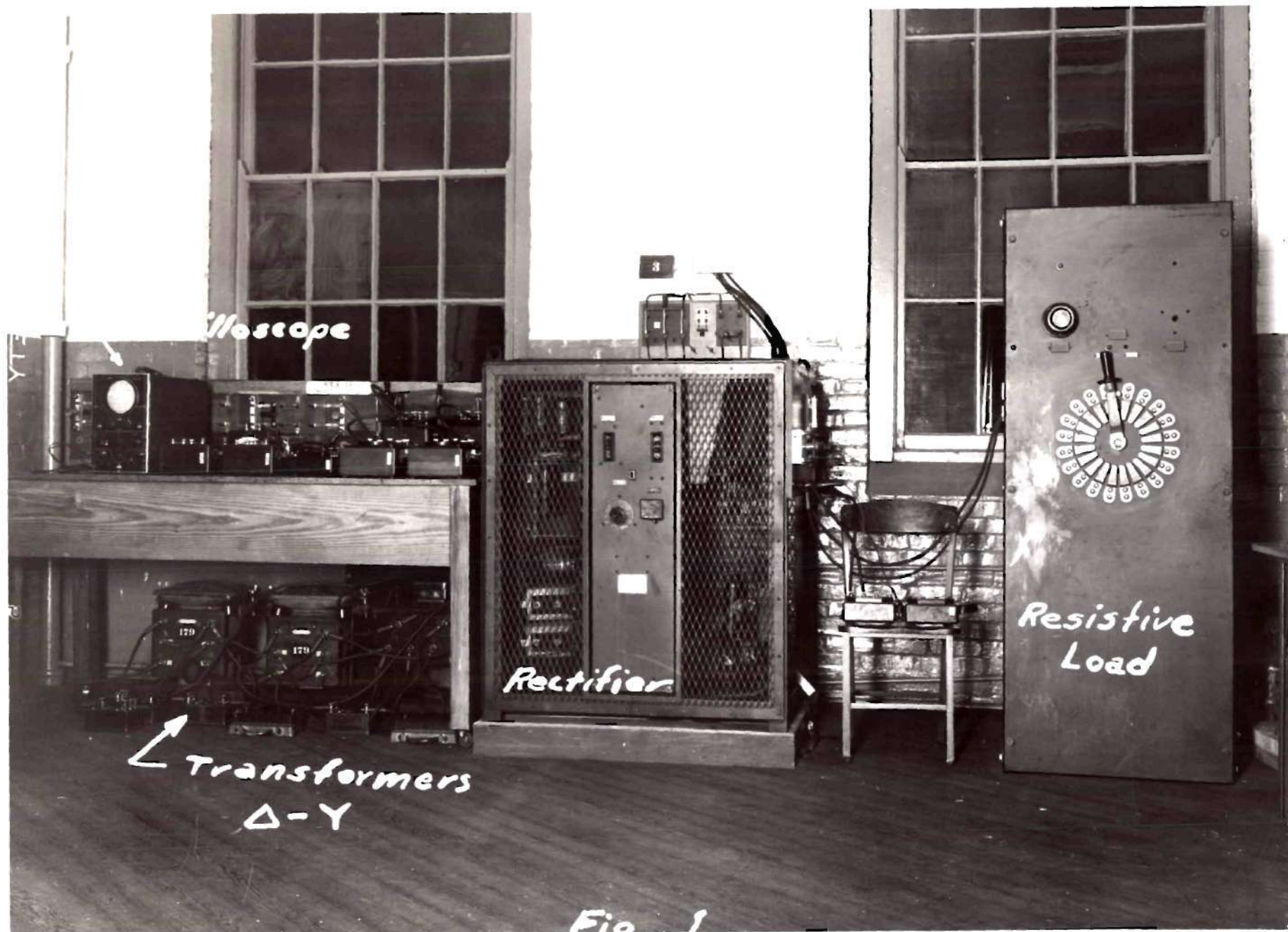
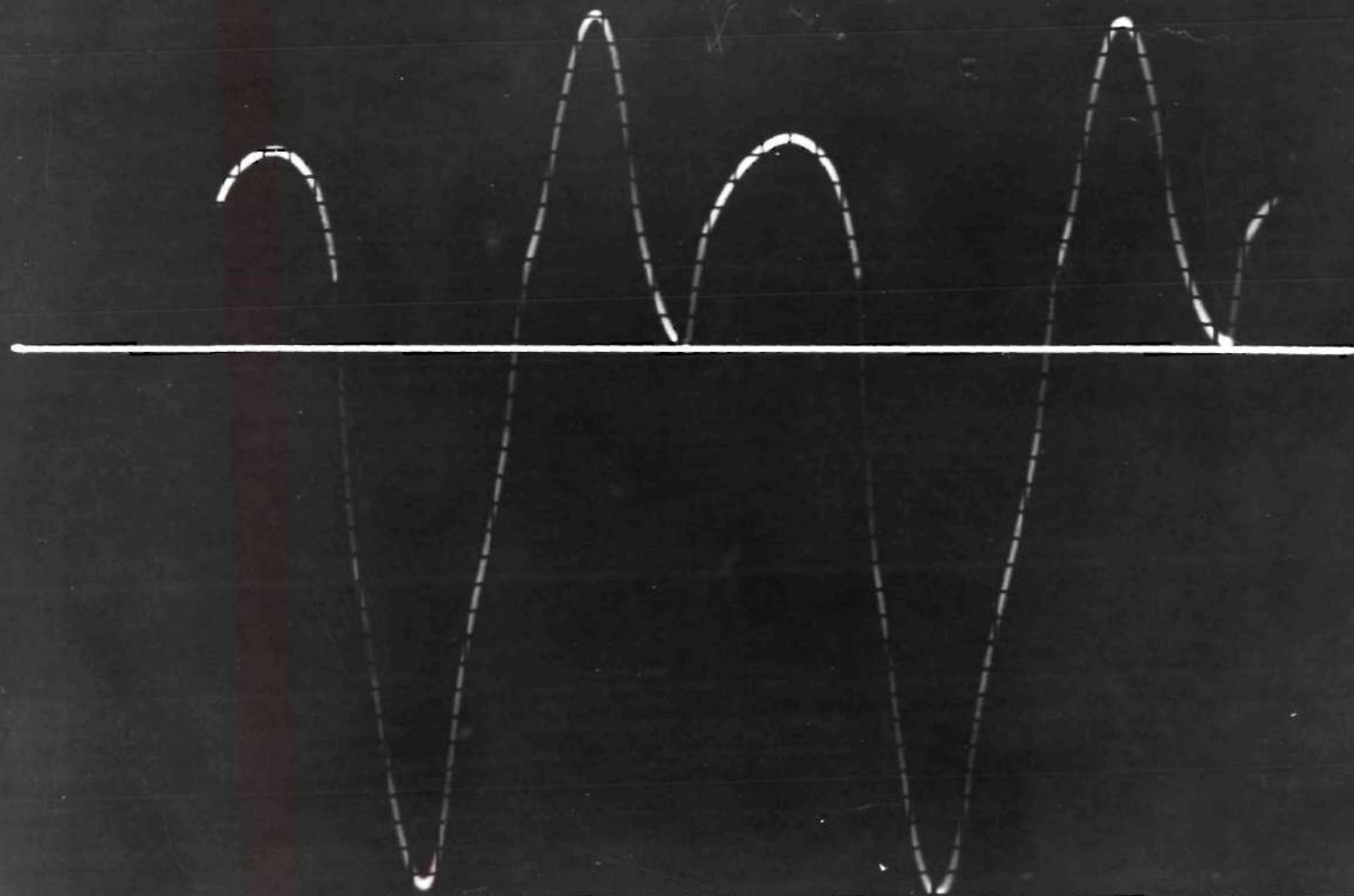


Fig. 1

PRIMARY LINE CURRENT

FULL LOAD

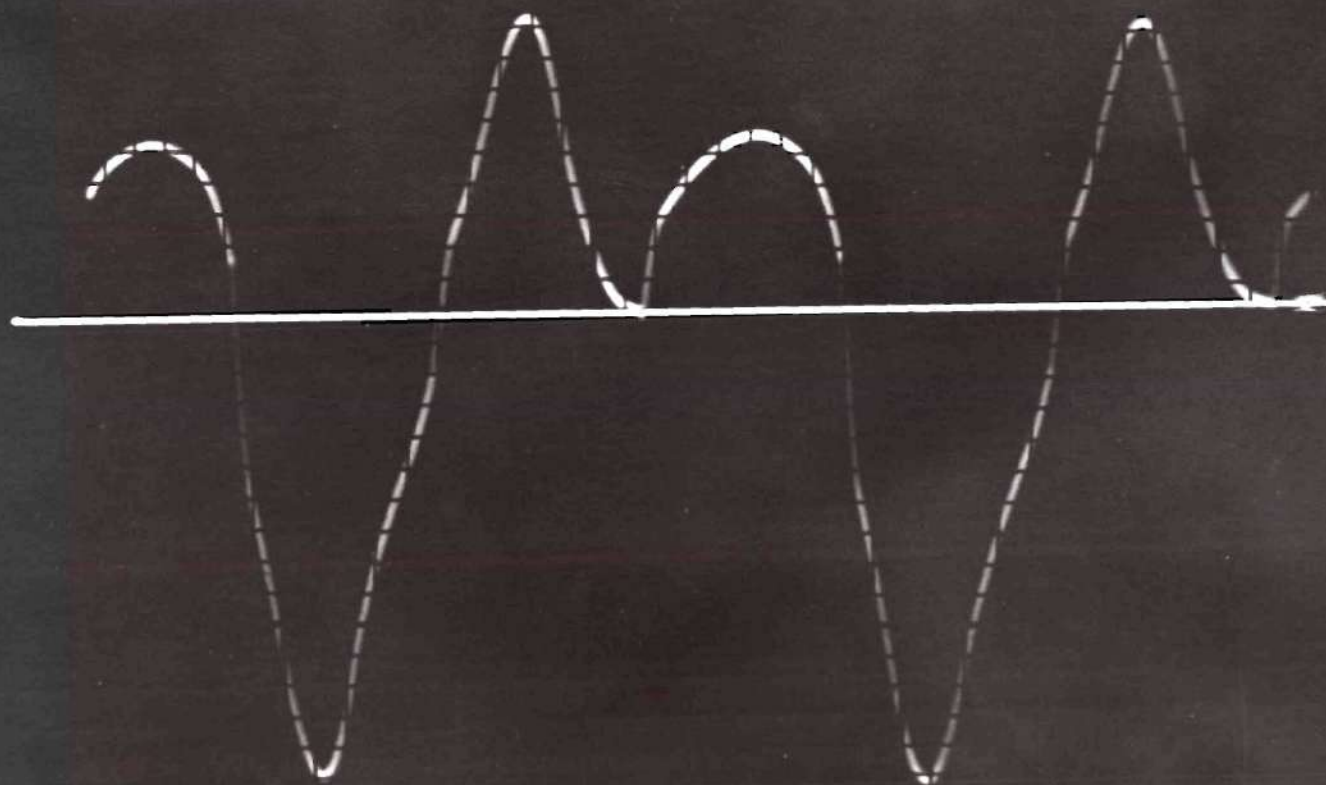
FIG. 2



PRIMARY LINE CURRENT

75% OF FULL LOAD

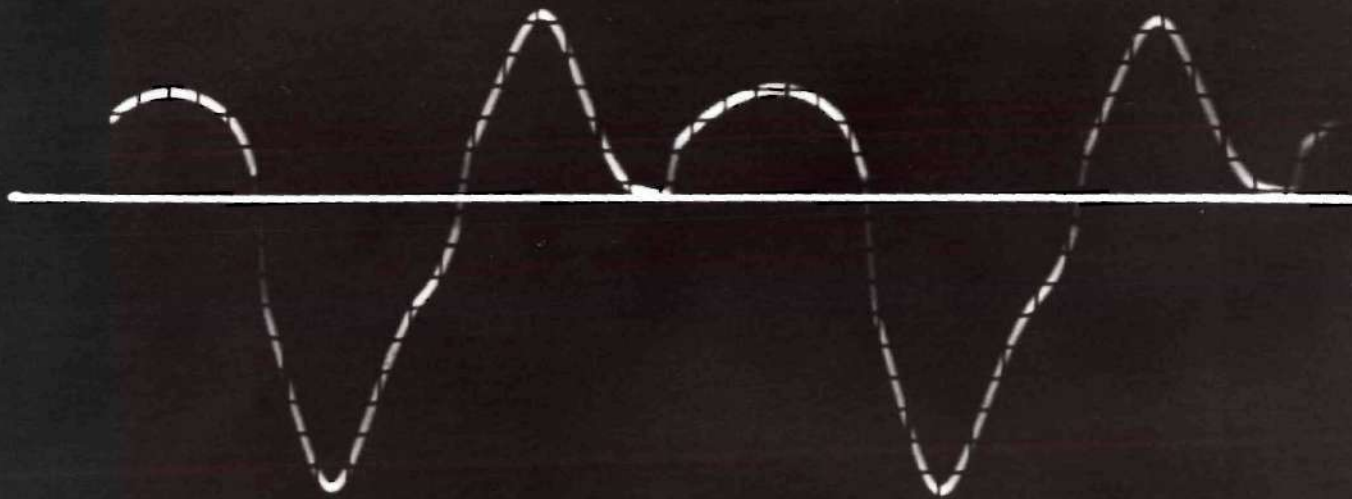
FIG. 3

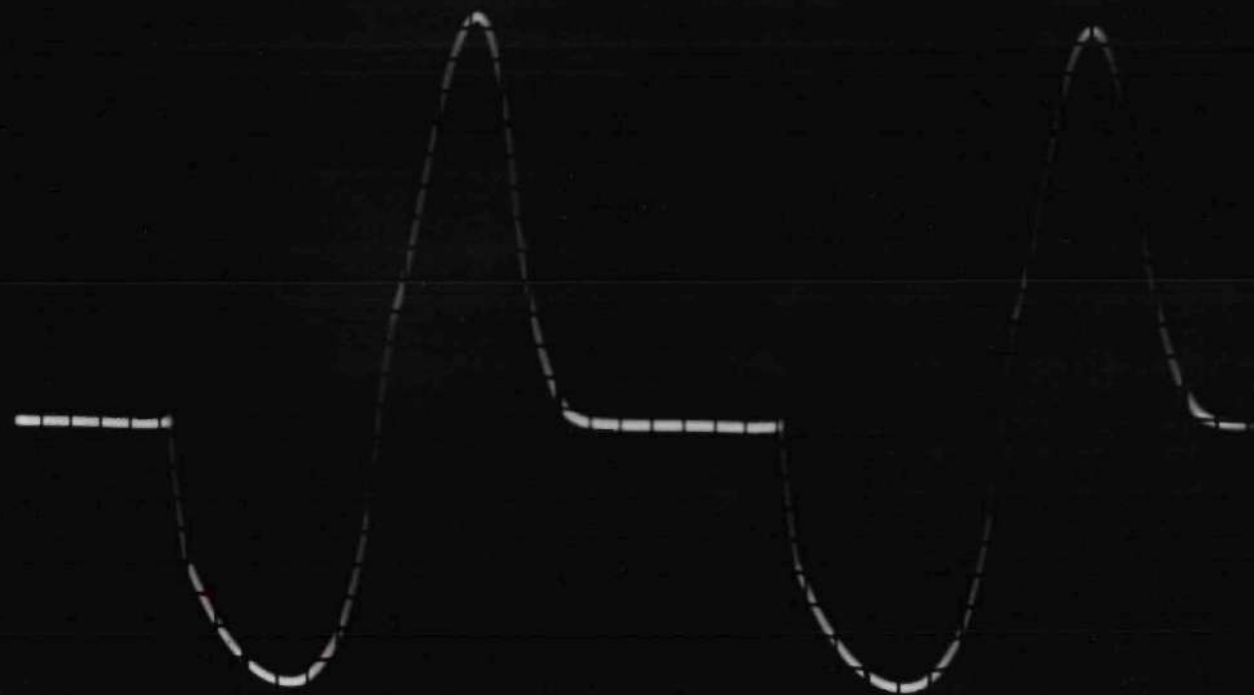


PRIMARY LINE CURRENT

46% FULL LOAD

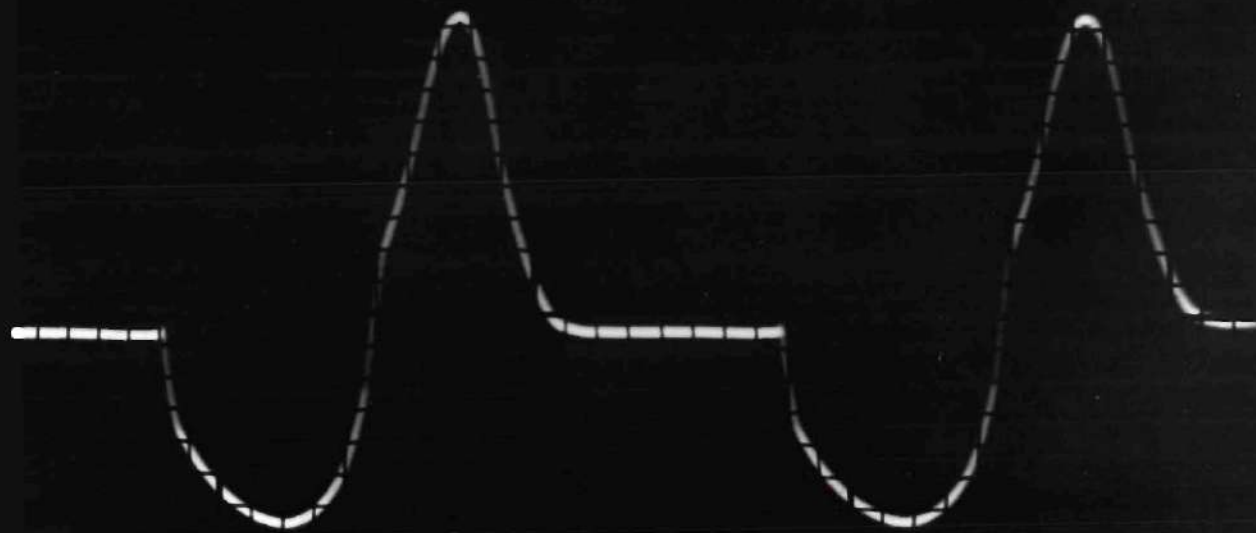
FIG. 4





Primary Phase Current
Full Load

Fig. 5



Primary Phase Current

75 % of Full Load

Fig. 6



Primary Phase Current

46 % of Full Load

Fig. 7



Secondary Phase Current
Full Load

Fig. 8

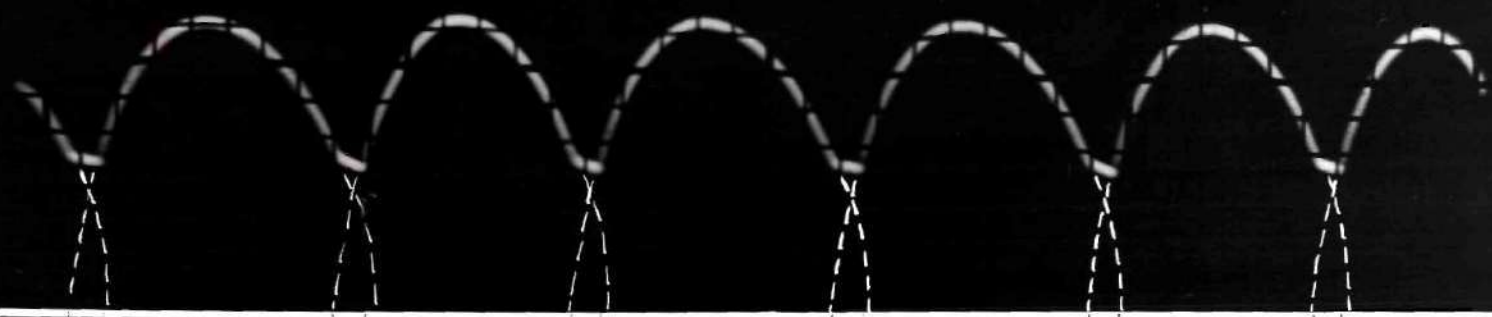


Secondary Phase Current
75 % of Full Load
Fig. 9



*secondary Phase Current
46 % of Full Load*

Fig. 10



Secondary Neutral Current
Full Load

Fig. 11

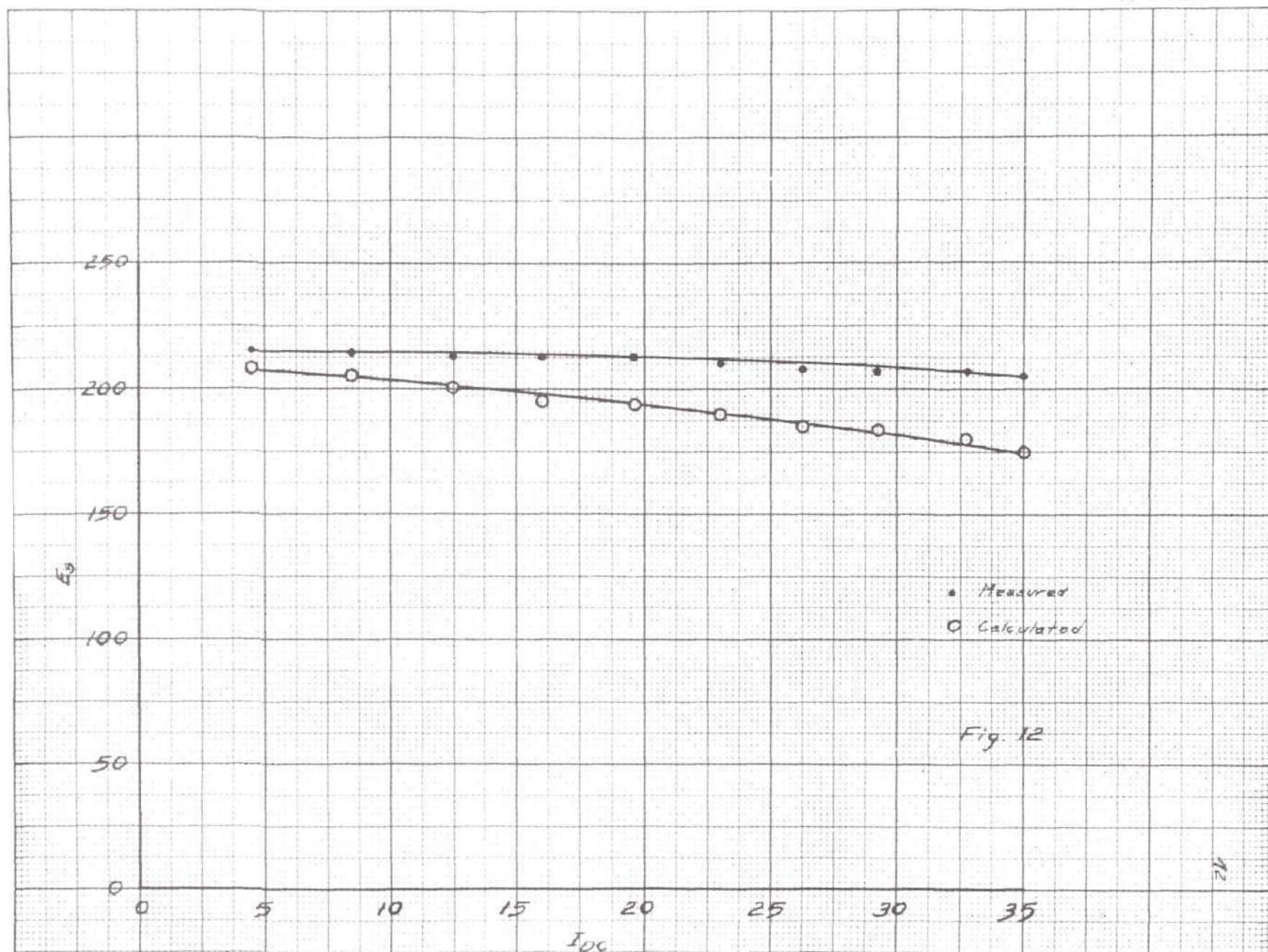
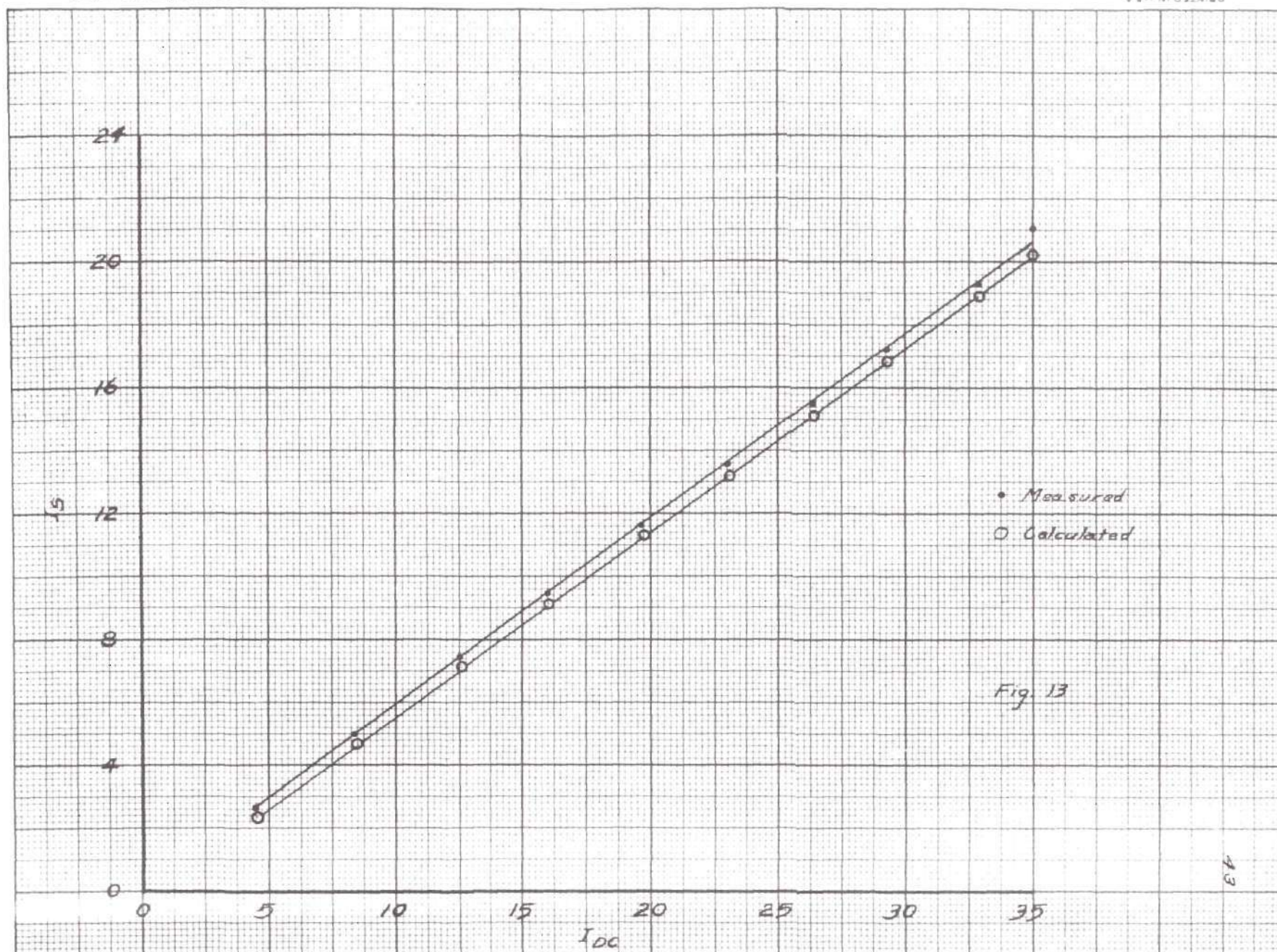
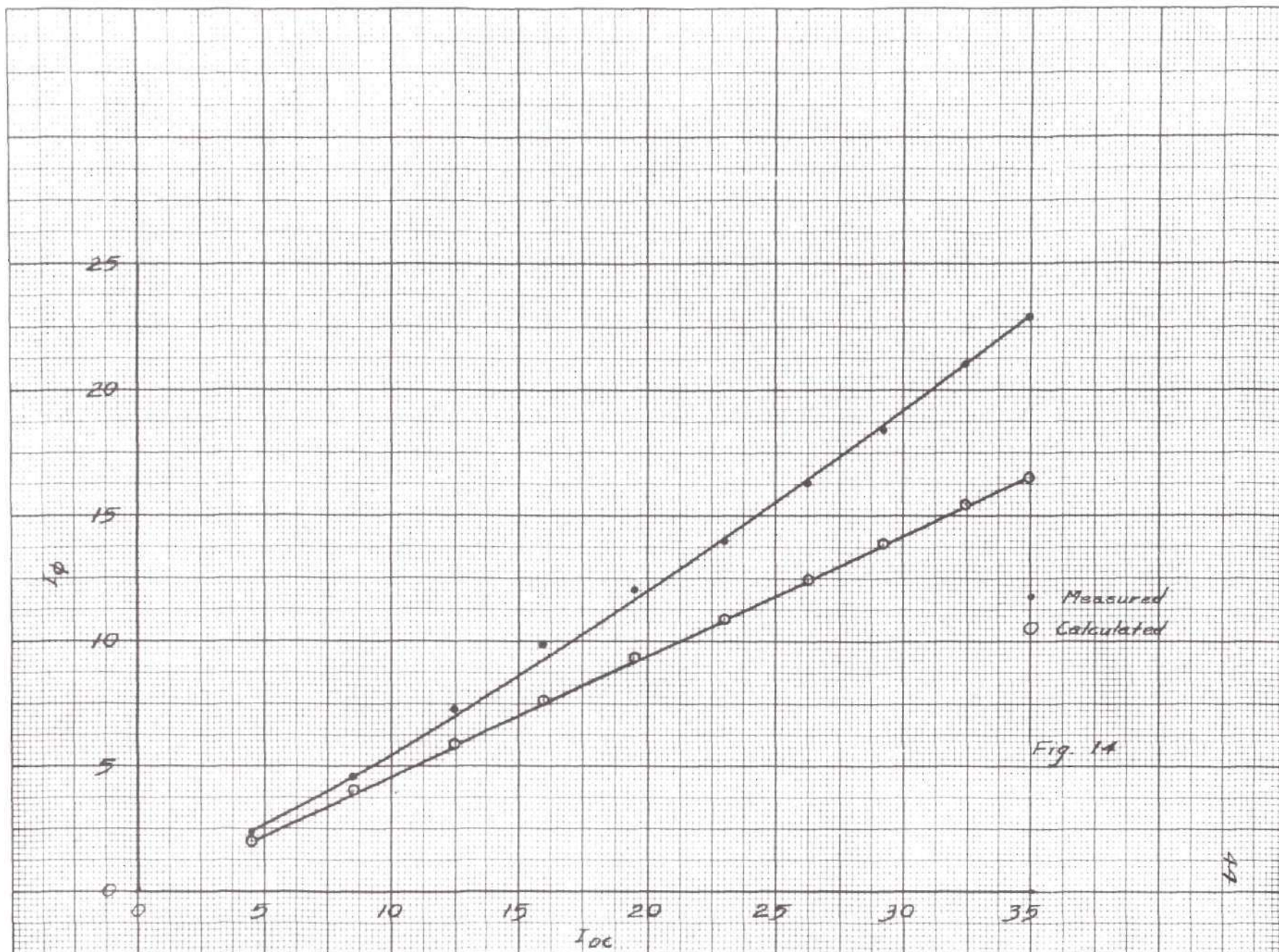
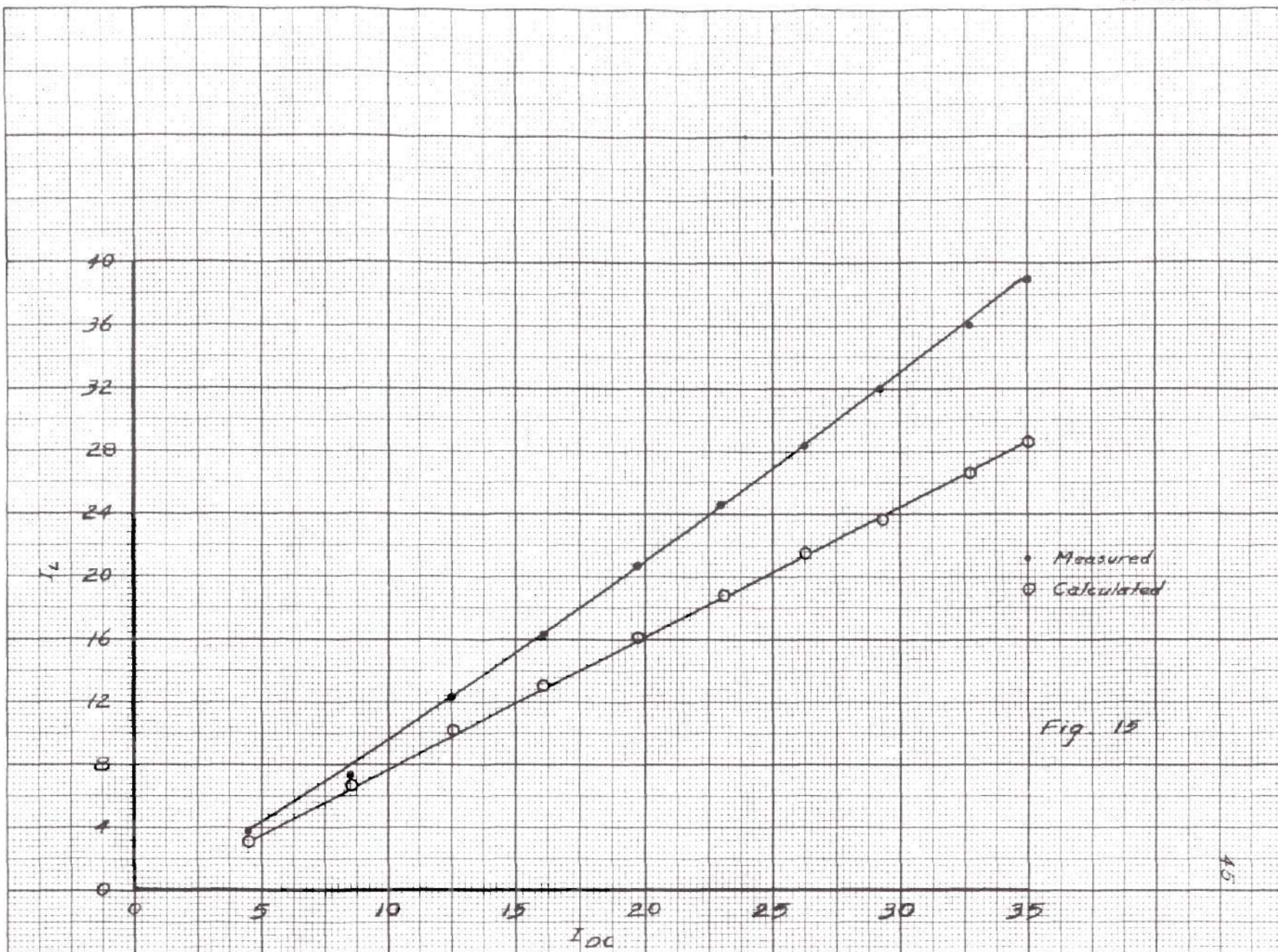


Fig. 12







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